1.13 Linear Buckling Analysis

Linear buckling analysis is used to determine the stability of structures subjected to compressive forces. The instability of a structure due to buckling is closely related to its geometric shape, stiffness, boundary conditions, and is independent of material strength. When a slender structure is subjected to axial compressive forces at its ends, it undergoes compressive deformation proportional to the magnitude of the load when the load is small. However, when the load exceeds a certain threshold, buckling occurs, causing significant deformation in the structure even without an increase in the load magnitude. Figure 1.13.1 illustrates the buckling shapes of a cylindrical column based on variations in its dimensions and length.

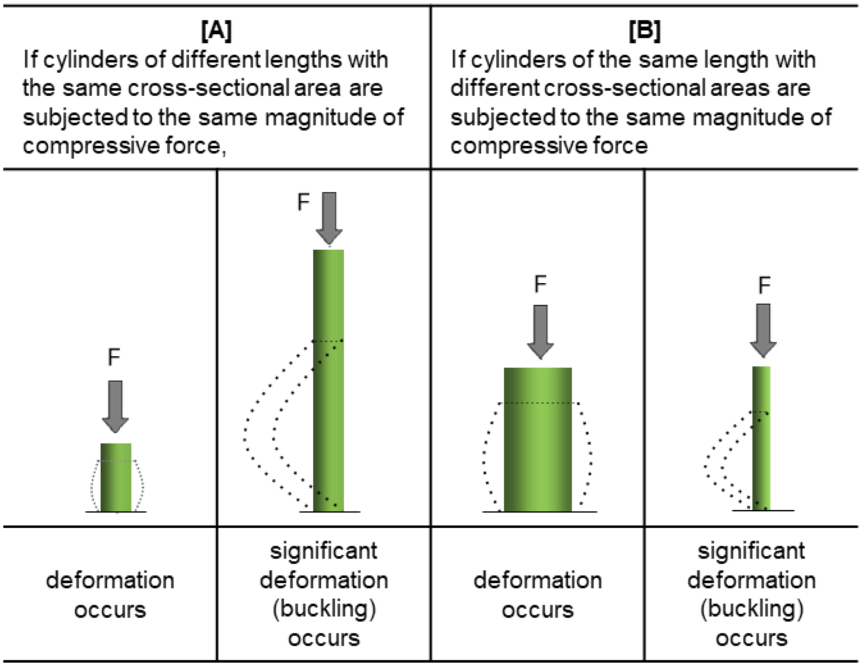


Figure 1.13.1 Buckling Shapes of Cylindrical Columns Based on Differences in Area and Length.

In the case of A, since the cross-sectional areas of both cylinders are the same, the stress magnitude generated by the formula $\sigma = F/A$ is equal. However, when the length of the cylinder is increased, significant deformation (buckling) can occur.
In the case of B, both cylinders have the same length, but different cross-sectional areas. When the same compressive force is applied, the smaller cross-sectional area will experience a higher stress, leading to buckling. In other words, as the cross-sectional area of the cylinder decreases (resulting in higher compressive stress), and as the length of the cylinder increases, the probability of buckling occurring becomes higher.

The types of structures for which linear buckling analysis is expected to be performed include:

- Large structures subjected to significant compressive loads.
- Thin, slender cylindrical structures subjected to axial loads.
- Thin-walled cylindrical structures exposed to external pressure.
- Thin-walled structures with pressure applied to their boundaries (e.g., pressure vessels).
- Long and slender cantilever structures that receive transverse end loads on their upper surfaces.

1.13.1 Governing Equation

In linear static analysis, it is assumed that the structure is subjected to a load and then returns to its original state when the load is removed, provided the structure is in a stable equilibrium state. However, under certain load conditions, the structure can become unstable, and if the external load reaches the critical load that induces instability, deformation can continue to occur even if the load magnitude does not increase, resulting in buckling.

Buckling occurs when a structure subjected to compressive loads converts membrane strain energy into bending strain energy, and the compressive load at this point is referred to as the buckling load or critical load. Typically, the buckling load is calculated by performing an eigenvalue analysis, and the computed eigenvalue (λ) is a factor (buckling load factor) for the applied load (P_a). The actual buckling load is then calculated as ($P_{cr} = \lambda P_a$). Linear buckling analysis is performed using the following equation:

$$(\mathbf{K} + \lambda \mathbf{K}_G)\boldsymbol{\varphi} = \mathbf{0} \tag{1.13.1}$$

Here , $\mathbf{K}$: Elastic Stiffness Matrix

$\mathbf{K}_G$: Geometric Stiffness Matrix

Elastic stiffness is the stiffness determined by the material and shape of a structure, while geometric stiffness is the stiffness determined by the stresses generated in a structure after the application of loads. This means that when a structure is subjected to stress, its stiffness changes, indicating a direct relationship between the applied load and geometric stiffness. For example, when a compressive load is applied to a structure like a column, geometric stiffness reduces the overall stiffness of the structure. Conversely, when a tensile load is applied, geometric stiffness adds to the effect of elastic stiffness, leading to an increase in the overall stiffness of the structure.

Buckling analysis is the situation where the stiffness of the structure becomes zero when the effects of both elastic stiffness and geometric stiffness are considered. The coefficient values that satisfy this condition correspond to the buckling load factor. The buckling load factor represents the ratio of the applied load to the buckling load. When the value of this coefficient is greater than 1.0, the structure is considered stable, and when it is less than 1.0, it is considered unstable. If the coefficient value is negative, it means that the load acts in the opposite direction, and the absolute value of the coefficient becomes the structure's buckling load factor. Eigenvalues represent the buckling shapes of the structure, typically showing buckling occurring in the first mode, but in some cases, higher modes (2nd, 3rd, etc.) of buckling can also occur depending on the shape of the structure.

1.13.2 Linear Buckling Analysis Flowchart

The procedure for linear buckling analysis is similar to that of a typical linear static analysis. However, there are some differences that set it apart:

- ► When applying static loads, you can apply the actual load magnitude acting on the structure or use unit loads.
- ► The buckling load, obtained by multiplying the calculated eigenvalues by the applied load, remains the same regardless of the load applied. Therefore, for convenience, unit loads are often used. In such cases, the eigenvalues directly represent the buckling load.
- ► Linear buckling analysis combines the conditions of linear static analysis and eigenvalue analysis in a single analysis run.
- ► When reviewing the results of buckling analysis, it is necessary to consider a combination of factors, including the eigenvalue-based buckling load factor, maximum stresses from linear static analysis, and material failure criteria, to assess the stability of the structure.

1.13.3 Modeling

When modeling an analysis model for linear buckling analysis, it is essential to generate a sufficient number of elements to represent the buckling mode shapes accurately. Typically, it is recommended to perform linear static analysis first and review the range of stress values obtained from it. Then, you can use the model created for linear analysis to perform buckling analysis. You can use all the elements commonly used in static analysis, but when considering the element size, it's crucial to take the buckling mode shapes into account and determine the element size accordingly. Especially when using solid elements, be mindful of the number of elements in the thickness direction to adequately represent bending behavior.

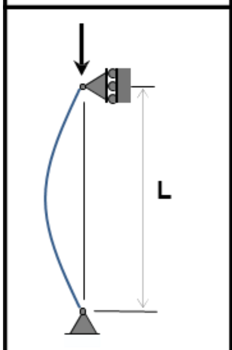
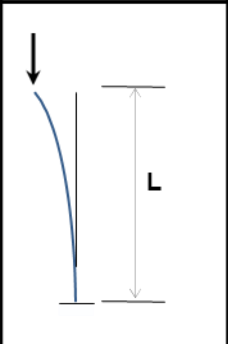
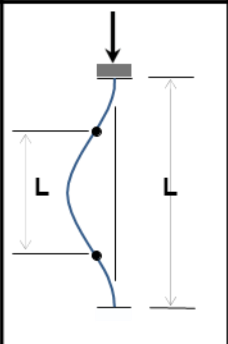
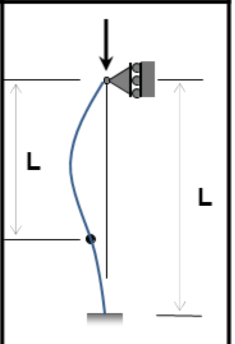
Linear buckling analysis can use all the elements and element properties commonly used in linear static analysis. The key is to adjust the element sizes to accurately capture the buckling mode shapes while considering the structure's stability and buckling behavior.

1.13.4 Material

In linear buckling analysis, the equation used to calculate eigenvalues does not include inertial forces, so there is no need to input density as a material property, unlike in modal analysis. Therefore, it is sufficient to input the material properties used in linear static analysis, which are the elastic modulus and Poisson's ratio.

1.13.5 Boundary and Load condition

The loads in linear buckling analysis are typically compressive loads acting along the axis of slender and long structures, and the method for applying these loads is the same as in linear static analysis. Similarly, the boundary conditions in linear buckling analysis are input in the same way as in linear static analysis. However, it's important to note that the buckling load of the structure can vary significantly based on the constraints of the components. Therefore, it's essential to reasonably set the constraints, taking into account the buckling behavior of the structure.

Pin both sides	Top free, bottom fixed	Fixed both sides	Top pin, bottom fixed
$P_{cr} = \frac{\pi^2 EI}{L^2}$	$P_{cr} = \frac{\pi^2 EI}{4L^2}$	$P_{cr} = \frac{4\pi^2 EI}{L^2}$	$P_{cr} = \frac{2.046\pi^2 EI}{L^2}$
			
$L_e = L$	$L_e = L$	$L_e = L$	$L_e = L$
$K = 1$	$K = 1$	$K = 1$	$K = 1$

1.13.6 Analysis Execution

Once the finite element model for the analysis subject is completed, and the load and boundary conditions are specified, you are ready to execute the analysis. When selecting the analysis type as linear buckling analysis, subcases for linear static analysis and mode analysis are automatically generated.

In linear buckling analysis, you typically don't need to calculate a large number of eigenvalues. Specifying the number of eigenvalues is usually sufficient if it's set to around 5 or fewer. However, when defining a range for eigenvalues, you should be cautious, as the values of the eigenvalues in linear buckling analysis can be influenced by the magnitude of the loads applied in static analysis.

So, in summary, linear buckling analysis is usually not concerned with obtaining a large number of eigenvalues, and you can often specify a small number of them (e.g., 5 or fewer). When specifying an eigenvalue range, be mindful of the fact that eigenvalues in linear buckling analysis may be affected by the magnitude of the loads applied in static analysis.

The method for specifying the number of eigenvalues in midas NFX is the same as shown in Figure 1.13.2, steps ① to ④.

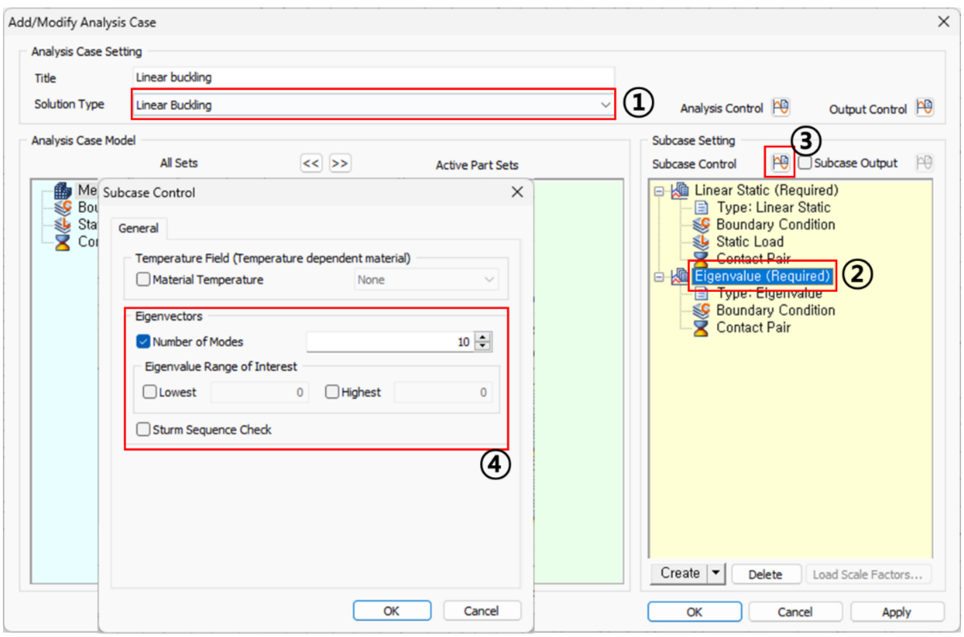


Figure 1.13.2 The method for specifying the number of eigenvalues

1.13.7 Analysis result

By analyzing the results of linear buckling analysis, user can anticipate four different scenarios. Each of these scenarios can be categorized based on the applied loads, the results of static analysis (maximum stress, σ_{max}), the results of eigenvalue analysis (buckling load factor, λ), and the material's ultimate strength (S_f).

Table 1.12.1 Method for analysis the result of bucking analysis

Condition	Analysis and Assessment
$\lambda \leq 1.0$ $\sigma_{max} < S_f$	Indicative of structural linear instability with suspected failure due to linear buckling.
$\lambda \geq 1.0$ $\sigma_{max} < S_f$	Indicative of structural linear stability
$\lambda \leq 1.0$ $\sigma_{max} \geq S_f$	It signifies structural linear or nonlinear instability. If λ is less than 1.0 and σ_{max} is similar to S_f, the structure is considered more susceptible to linear buckling. Conversely, if λ is close to 1.0 and σ_{max} has a much larger value than S_f, buckling phenomenon is likely to be nonlinear. In such cases, a more detailed review is performed by conducting nonlinear static analysis and buckling analysis.
$\lambda \geq 1.0$ $\sigma_{max} \geq S_f$	In such cases, the structure may exhibit linear stability, but the presence of nonlinear material behavior necessitates the need for nonlinear static analysis.

When performing linear buckling analysis using unit loads, it is essential to review the stresses induced in the structure by applying the calculated buckling loads. In linear buckling analysis, the first step is to examine the buckling factor (eigenvalue), which is different from the natural frequencies (in Hz) obtained in modal analysis. If, for instance, a static analysis used a load of -2,000N, then the actual buckling load for each mode is obtained by multiplying the eigenvalue by this load. If the buckling factor for the first mode is 0.643, which is less than 1.0, it signifies that the structure will buckle when subjected to a compressive load of -1,286N.

If the -2,000N load represents the actual applied load on the structure, then it suggests a problem with the structure's linear stability, and linear buckling-induced failure is anticipated. In this case, the buckling shape at failure can be assumed to be similar to the mode shape of the first mode.